

NAG Toolbox for MATLAB

f08kr

1 Purpose

f08kr computes the singular value decomposition (SVD) of a complex m by n matrix A , optionally computing the left and/or right singular vectors, by using divide-and-conquer method.

2 Syntax

```
[a, s, u, vt, info] = f08kr(jobz, a, 'm', m, 'n', n)
```

3 Description

The SVD is written as

$$A = U\Sigma V^H,$$

where Σ is an m by n matrix which is zero except for its $\min(m, n)$ diagonal elements, U is an m by m unitary matrix, and V is an n by n unitary matrix. The diagonal elements of Σ are the singular values of A ; they are real and nonnegative, and are returned in descending order. The first $\min(m, n)$ columns of U and V are the left and right singular vectors of A .

Note that the function returns V^H , not V .

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: <http://www.netlib.org/lapack/lug>

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **jobz** – string

Specifies options for computing all or part of the matrix U .

jobz = 'A'

All m columns of U and all n rows of V^H are returned in the arrays **u** and **vt**.

jobz = 'S'

The first $\min(m, n)$ columns of U and the first $\min(m, n)$ rows of V^H are returned in the arrays **u** and **vt**.

jobz = 'O'

If $m \geq n$, the first n columns of U are overwritten on the array **a** and all rows of V^H are returned in the array **vt**. Otherwise, all columns of U are returned in the array **u** and the first m rows of V^H are overwritten in the array **vt**.

jobz = 'N'

No columns of U or rows of V^H are computed.

Constraint: **jobz** = 'A', 'S', 'O' or 'N'.

2: **a(lda,*) – complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The m by n matrix A .

5.2 Optional Input Parameters

1: **m – int32 scalar**

Default: The first dimension of the array **a**.

m , the number of rows of the matrix A .

Constraint: $\mathbf{m} \geq 0$.

2: **n – int32 scalar**

Default: The second dimension of the array **a**.

n , the number of columns of the matrix A .

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldu, ldvt, work, lwork, rwork, iwork

5.4 Output Parameters

1: **a(lda,*) – complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If **jobz** = 'O', **a** is overwritten with the first n columns of U (the left singular vectors, stored column-wise) if $\mathbf{m} \geq \mathbf{n}$; **a** is overwritten with the first m rows of V^H (the right singular vectors, stored row-wise) otherwise.

If **jobz** $\neq$ 'O', the contents of **a** are destroyed.

2: **s(*) – double array**

Note: the dimension of the array **s** must be at least $\max(1, \min(\mathbf{m}, \mathbf{n}))$.

The singular values of A , sorted so that $\mathbf{s}(i) \geq \mathbf{s}(i+1)$.

3: **u(ldu,*) – complex array**

The first dimension, **ldu**, of the array **u** must satisfy

if **jobz** = 'S' or 'A' or **jobz** = 'O' and $\mathbf{m} < \mathbf{n}$, $\mathbf{ldu} \geq \max(1, \mathbf{m})$;
 $\mathbf{ldu} \geq 1$ otherwise.

The second dimension of the array must be at least $\max(1, ucol)$, where $ucol$ is the number of columns of U requested

$ucol = \mathbf{m}$ if **jobz** = 'A' or **jobz** = 'O' and $\mathbf{m} < \mathbf{n}$; $ucol = \min(\mathbf{m}, \mathbf{n})$ if **jobz** = 'S'.

If **jobz** = 'A' or **jobz** = 'O' and $\mathbf{m} < \mathbf{n}$, **u** contains the m by m unitary matrix U .

If **jobz** = 'S', **u** contains the first $\min(m, n)$ columns of U (the left singular vectors, stored column-wise).

If **jobz** = 'O' and $\mathbf{m} \geq \mathbf{n}$, or **jobz** = 'N', **u** is not referenced.

4: **vt(ldvt,*) – complex array**

The first dimension, **ldvt**, of the array **vt** must satisfy

if **jobz** = 'A' or **jobz** = 'O' and $\mathbf{m} \geq \mathbf{n}$, $\mathbf{ldvt} \geq \max(1, \mathbf{n})$;
 if **jobz** = 'S', $\mathbf{ldvt} \geq \max(1, \min(\mathbf{m}, \mathbf{n}))$;
 $\mathbf{ldvt} \geq 1$ otherwise.

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If **jobz** = 'A' or **jobz** = 'O' and $\mathbf{m} \geq \mathbf{n}$, **vt** contains the n by n unitary matrix V^H .

If **jobz** = 'S', **vt** contains the first $\min(m, n)$ rows of V^H (the right singular vectors, stored row-wise).

If **jobz** = 'O' and $\mathbf{m} < \mathbf{n}$, or **jobz** = 'N', **vt** is not referenced.

5: **info – int32 scalar**

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **jobz**, 2: **m**, 3: **n**, 4: **a**, 5: **lda**, 6: **s**, 7: **u**, 8: **ldu**, 9: **vt**, 10: **ldvt**, 11: **work**, 12: **lwork**, 13: **rwork**,
 14: **iwor**k, 15: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

f08kr did not converge, the updating process failed.

7 Accuracy

The computed singular value decomposition is nearly the exact singular value decomposition for a nearby matrix $(A + E)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the *machine precision*. In addition, the computed singular vectors are nearly orthogonal to working precision. See Section 4.9 of Anderson *et al.* 1999 for further details.

8 Further Comments

The total number of floating-point operations is approximately proportional to mn^2 when $m > n$ and m^2n otherwise.

The singular values are returned in descending order.

The real analogue of this function is f08kd.

9 Example

```
jobz = 'Overwrite A by V**H';
a = [complex(0, +0), complex(-0.98, -1.98), complex(0.62, +0.46),
```

```

complex(-0.37, ...
-0.38), complex(0.83, -0.51), complex(1.08, +0.28);
    complex(0, +0), complex(-1.2, -0.19), complex(1.01, -0.02),
complex(0.19, ...
+0.54), complex(0.2, -0.01), complex(0.2, +0.12);
    complex(0, +0), complex(-0.66, -0.42), complex(0.63, +0.17),
complex(-0.98, ...
+0.36), complex(-0.17, +0.46), complex(-0.070000000000000001, -1.23);
    complex(0, +0), complex(-0.810000000000000001, -0.560000000000000001),
...
    complex(-1.11, -0.6), complex(0.22, +0.2), complex(1.47, -1.59),
complex(0.26, -0.26)];
[aOut, s, u, vt, info] = f08kr(jobz, a)

```

```

aOut =
    Columns 1 through 4
    0.0000 + 0.0000i   -0.3233 - 0.7034i    0.1445 + 0.0453i   -0.1596 -
0.0295i
    0.0000 - 0.0000i    0.0032 - 0.1689i    0.0278 + 0.6309i   -0.2002 -
0.2784i
    -0.0000 + 0.0000i    0.2192 - 0.1445i   -0.4367 - 0.0070i   -0.4944 +
0.0989i
    0.0000 - 0.0000i    0.0822 + 0.1707i   -0.2149 + 0.2811i    0.1419 +
0.6596i
    Columns 5 through 6
    0.4588 - 0.1374i    0.3490 + 0.0153i
   -0.6182 - 0.1634i    0.2053 - 0.0638i
    0.1108 + 0.0403i   -0.2256 - 0.6490i
    0.0497 + 0.1586i    0.5933 - 0.0641i
s =
    3.5551
    2.7026
    1.4892
    0.2650
u =
    0.7658           0.2193           -0.2761           -0.5379
    0.2208 - 0.2391i   -0.0863 - 0.3034i   -0.5495 - 0.2629i    0.5613 -
0.3292i
    0.1600 - 0.1855i    0.1419 - 0.5253i    0.6538 - 0.0326i   -0.0500 -
0.4615i
    0.3957 - 0.3018i   -0.3686 + 0.6484i    0.3440 - 0.0768i    0.2365 -
0.1259i
vt =
    Columns 1 through 4
   -0.0000 + 0.0000i    0.0000 + 0.0000i    0.0000 + 0.0000i    0.0000 +
0.0000i
    Columns 5 through 6
    0.0000 + 0.0000i    0.9800
info =
    0

```